Validation of CFD piston wave generation against classical wavemaker theory and the Madsen (1970) start-up benchmark

OpenFOAM moving-paddle flume vs the Biésel / Ursell-Dean-Yu transfer function and Madsen's wave-by-wave laboratory record of the starting transient, with an active-absorption reflection check

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Technical report, v1.0 - August 2026

DOI 10.5281/zenodo.21858419 (Zenodo)

Companion report (CFD tier, propagation and breaking): DOI 10.5281/zenodo.21776521 (Zenodo)

Companion report (wave-model tier, basin scale): DOI 10.5281/zenodo.21804625 (Zenodo)

Abstract. *In a surf pool every wave starts at the generator, so the first thing a wave-modelling toolchain must get right is the generator itself: how much wave a given paddle motion delivers, and what happens in the first seconds after the machine starts. This report validates the moving-boundary piston wavemaker of our OpenFOAM toolchain against the classical benchmarks of wavemaker theory. A 2D numerical flume reproduces the laboratory case documented by Madsen (1970): depth 0.4572 m, period 1.16 s, piston stroke 100.6 mm, h/L = 0.240, wave steepness H/L = 0.074. Three results.* ***Steady transfer function:*** *the delivered wave height at the paddle is 100.5% of the Biésel first-order prediction confirmed experimentally by Ursell, Dean & Yu (1960).* ***Starting transient:*** *compared wave by wave against Madsen's Table 1 at his measurement station 45 ft (13.716 m) from the paddle, using his own definitions, the leading-packet overshoot is 1.22x the final periodic wave height in the CFD vs 1.22x in his laboratory run and 1.20x in his published chart record, digitised here - while the linearised transient theory he evaluated (Kennard) predicts 1.13x. Across the three largest waves of the packet the CFD tracks the laboratory within 0.02 in* H/H_p *units. The digitised chart record independently agrees with his tabulated run to 0.03 rms, validating the digitisation and confirming his repeatability claim.* ***Reflection:*** *a Goda-Suzuki two-gauge decomposition gives a reflection coefficient of 2.7% for the active-absorption far boundary, so the comparisons are not contaminated by re-reflection. Honest limits are quantified: numerical amplitude decay to 80% of ideal-fluid theory after 7.2 wavelengths at this steepness (the leaky laboratory paddle delivered 65-71%; an ideal fluid, 100% - the CFD sits between, as it should with no leakage), spatial modulation by the free second harmonic that Madsen's own second-order analysis predicts, a single tested kh, and a soft-start ramp that differs from the laboratory's abrupt start on the rise flank of the packet.*

1. Introduction

The wave budget of a surf pool is set at the generator. The transfer function - how much wave height a given paddle stroke delivers at a given depth and period - feeds directly into energy per wave, machinery sizing and cycle economics; get it wrong by ten per cent and every downstream number inherits the error. The start-up matters too: wavemaker theory and laboratory practice agree that a paddle starting from rest does not simply grow into its steady wave - the leading group overshoots, and the largest wave of a burst arrives a few periods after the front, some 20% higher than the periodic state. Madsen (1970) described this 'large first wave' as the response of a slightly damped mechanical system to an exciting force. A toolchain that will be trusted for generation design must reproduce both behaviours from first principles.

This report closes the generation link of our validation chain. The companion CFD-tier report validates propagation and breaking against two wave-flume benchmarks (Zvarich 2026a); the companion wave-model report validates basin-scale refraction-diffraction on the Berkhoff shoal (Zvarich 2026b). Here the moving boundary itself is validated: the paddle is not a velocity-inlet surrogate but a mesh-moving piston, driven exactly as a physical machine would be, and judged against the classical theory (Biésel & Suquet 1951; Ursell, Dean & Yu 1960; Kennard 1949) and against Madsen's laboratory record. The same two principles as in the companions were fixed in advance: **no tuning to fit -** the case runs the published parameters, and nothing was adjusted to chase the measurements; and **divergence is a result -** where the CFD, the theory and the laboratory disagree, the disagreement is reported with its mechanism, because that is what design margins are built from.

2. The benchmark

Two classical anchors are combined in one case. For the **steady state**, first-order wavemaker theory (Havelock 1929; Biésel & Suquet 1951) gives the piston transfer function $H/S = m_1 = 2(\cosh 2kh - 1)/(\sinh 2kh + 2kh)$; Ursell, Dean & Yu (1960) confirmed it experimentally to within a few per cent once paddle leakage was controlled, and it has been the design formula for piston wavemakers since. At kh = 1.508 the theory gives $m_1 = 1.399$: a 100.6 mm stroke should deliver a 140.7 mm wave.

For the **starting transient**, Kennard (1949) solved the linearised problem of a wavemaker starting from rest; Madsen (1970) evaluated that solution for a sinusoidal piston, ran the matching laboratory experiments in CERC's 72-foot tank, and published the comparison wave by wave. His Table 1 lists, for h/L = 0.240 at a gauge 45 ft from the paddle, the height of every wave as it arrives - experiment and linear computation side by side - and his Figure 1 shows the full chart record of the same case. That combination (tabulated per-wave data plus a published trace, at documented parameters) makes this the natural CFD benchmark for wave generation, the same way Beji-Battjes serves propagation. The case is deliberately non-trivial: at steepness H/L = 0.074 the waves are strongly second-order (measured second-harmonic content up to $a_2/a_1 = 0.24$), and Madsen himself flags the Benjamin-Feir side-band instability as active at this h/L - the linear transient theory is expected to underpredict the overshoot, and does.

Parameter	Value
Still-water depth	h = 0.4572 m (1.5 ft)
Wave period / paddle stroke	T = 1.16 s / S = 100.6 mm (0.33 ft)
Dispersion	kh = 1.508, L = 1.905 m, h/L = 0.240
Theoretical wave height	$m_1 S$ = 140.7 mm; steepness H/L = 0.074
Madsen's station	45 ft = 13.716 m from the paddle (7.2 wavelengths)
Regime	non-breaking; strongly second-order (a_2/a_1 up to 0.24)

3. Numerical set-up

OpenFOAM v2512, interFoam (volume-of-fluid, laminar), in a 2D flume 20 m long and 0.85 m high on a 340k-cell mesh (dx = 10 mm, dz = 5 mm). The paddle is the standard waveMaker piston boundary: the left wall physically translates (displacementLaplacian mesh motion) with the stroke computed by the solver's built-in Biésel formula from the target wave height, exactly as a laboratory machine is driven by its calibration; the motion ramps up over 2T = 2.32 s. The far end is an active absorber (shallowWaterAbsorption); its measured performance is Section 4.2. Five numerical wave gauges sample the surface at 50 Hz: x = 0.5 m (transfer function), a Goda pair at 8.00/8.25 m (reflection), Madsen's station at 13.716 m, and 16.0 m (Fig. 1). The run covers 28 s of physical time - about 15 wave periods at the far gauges after front arrival - and completes in 5.8 h on 8 cores of an Apple M3 Pro; like the companion benchmarks, laptop-scale.

Two set-up honesty notes. The initial water fill is quantised to the 5 mm cell height, so the still-water level is 0.4550 m rather than the nominal 0.4572 m; propagating this through the dispersion relation changes the theoretical m_1 by 0.3% - negligible against the effects studied, and the nominal depth is used throughout. And the start protocol differs from the laboratory: Madsen's machine started abruptly from its mean position at full amplitude, while this run ramps over two periods. The consequence is visible exactly where expected - on the rise flank of the transient packet (Section 4.3) - and does not affect the steady state.

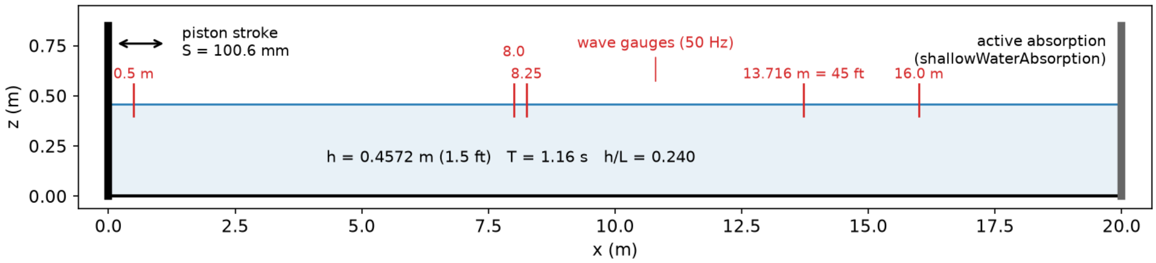

Figure 1. Numerical flume: mesh-moving piston paddle (left), five surface gauges, active absorption (right). Depth, period and stroke follow Madsen (1970), fig. 1 case: h/L = 0.240.

4. Results

4.1 Steady transfer function: 100.5% of theory

Over the final eight seconds of the run the mean wave height at the paddle gauge (x = 0.5 m) is 141.4 mm against the first-order prediction of 140.7 mm: the moving-boundary generator delivers **100.5%** of the Biésel / Ursell-Dean-Yu transfer function (Fig. 2). This is the number a generation design leans on when it converts a stroke and a period into a wave, and it is reproduced here from first principles with no calibration knob: the only inputs are the paddle motion and the fluid.

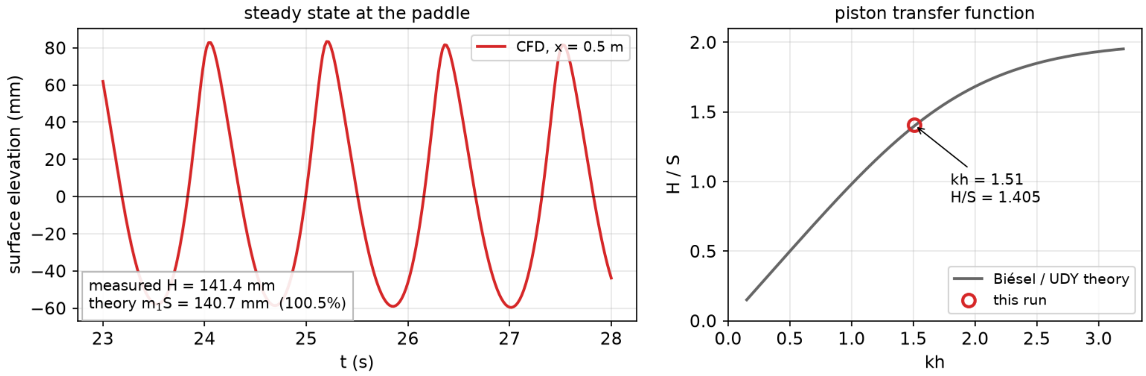

Figure 2. Left: steady surface record at the paddle gauge; measured mean height 141.4 mm vs theoretical 140.7 mm. Right: the piston transfer function H/S = m1(kh) with this run's point: 100.5% of theory at kh = 1.508.

4.2 Reflection check: K_r = 2.7%

Before trusting any downstream comparison, the far boundary must be shown not to send the waves back. A Goda-Suzuki (1976) two-gauge decomposition on the 8.00/8.25 m pair (spacing 0.25 m = 0.825 rad of phase, inside the method's valid window; analysis over 8 periods, t = 18-27.3 s) gives an incident amplitude of 61.1

mm and a reflected amplitude of 1.6 mm: **K_r = 2.7%**. The active absorber works, and the transient and steady-state comparisons below are clean of re-reflection at the 3% level.

4.3 The starting transient, wave by wave against Madsen

Madsen's definitions are adopted verbatim: wave height is the surface-elevation difference between a trough and the preceding crest (zero-down-crossing), and the final periodic height H_p is the mean of the burst excluding the large leading waves - here, the mean over all waves after the two largest (for the CFD, H_p = 118.7 mm on that protocol). Table 1 and Figure 3 compare four series at his 45 ft station: his laboratory run, his published chart record (Fig. 1 of the paper) digitised for this report, the linearised Kennard-type computation he published alongside, and the present CFD.

Wave No. (as arrived)	1	2	3	4	5	6	7	8	9	10	11
Laboratory (Madsen Table 1)	0.14	0.21	0.35	0.52	0.83	**1.22**	1.13	0.91	1.10	0.94	1.05
Laboratory (Fig. 1, digitised here)	0.13	0.20	0.32	0.49	0.76	**1.20**	1.11	0.85	1.09	0.95	1.02
Linear transient theory (Kennard)	0.11	0.18	0.29	0.45	0.69	0.95	**1.13**	1.12	0.92	0.96	1.02
CFD, this report	0.04	0.08	0.18	0.36	0.86	**1.22**	1.11	1.15	0.97	1.18	1.06

Table 1. Wave-by-wave height of the starting transient at 45 ft (13.716 m), normalised by the final periodic height H_p of each series (Madsen's protocol). Bold: the largest wave of each series. Series aligned at the largest wave; absolute arrival numbering differs between start protocols (ramp vs abrupt).

The overshoot lands on the laboratory value. The largest wave of the burst reaches 1.22x the periodic height in the CFD (144.5 mm crest-to-trough at t = 15.6 s) vs 1.22x in Madsen's tabulated run and 1.20x in his chart record - while the linear theory gives 1.13x. Across the three peak waves (5-7) the CFD tracks the laboratory within 0.02 in H/H_p units. The nonlinear amplification of the leading wave that the linearised theory misses - Madsen attributed it to the second-order effects his steepness range made unavoidable - is reproduced by the CFD, which needs no such correction: the steady crest/trough asymmetry at the station is 1.23, squarely second-order.

Two honest divergences, both with mechanisms. On the **rise flank** (waves 1-4) the CFD grows later and steeper: its paddle ramps over two periods while Madsen's machine started abruptly, so the front of the numerical burst is softer - the first tabulated laboratory wave is already 0.14 H_p where the CFD's is 0.04. This is a start-protocol difference, not a fluid-dynamics one. In the **ring-down** (waves 8-10) the phase of the decaying oscillation differs: the laboratory dips at wave 8 and recovers at 9, the CFD holds high through 8 and dips at 9. Madsen flags the Benjamin-Feir side-band instability as active at this h/L in the laboratory tail; at 7.2 wavelengths in a clean 2D simulation the same instability has had far less noise to grow from. Including this ring-down window, the rms difference between CFD and laboratory over waves 4-9 is 0.13 in H/H_p units; the sensitivity of the headline overshoot to the H_p averaging window is 1.22-1.28 (taking H_p from the final 8 s alone, 112.6 mm, gives the upper value).

The digitisation itself is cross-validated: over the eleven tabulated waves, the per-wave heights read off the published chart record agree with Madsen's tabulated run to 0.03 rms (max 0.07) in H/H_p units - two laboratory realisations he described as 'practically identical' indeed collapse onto each other, and onto the digitisation.

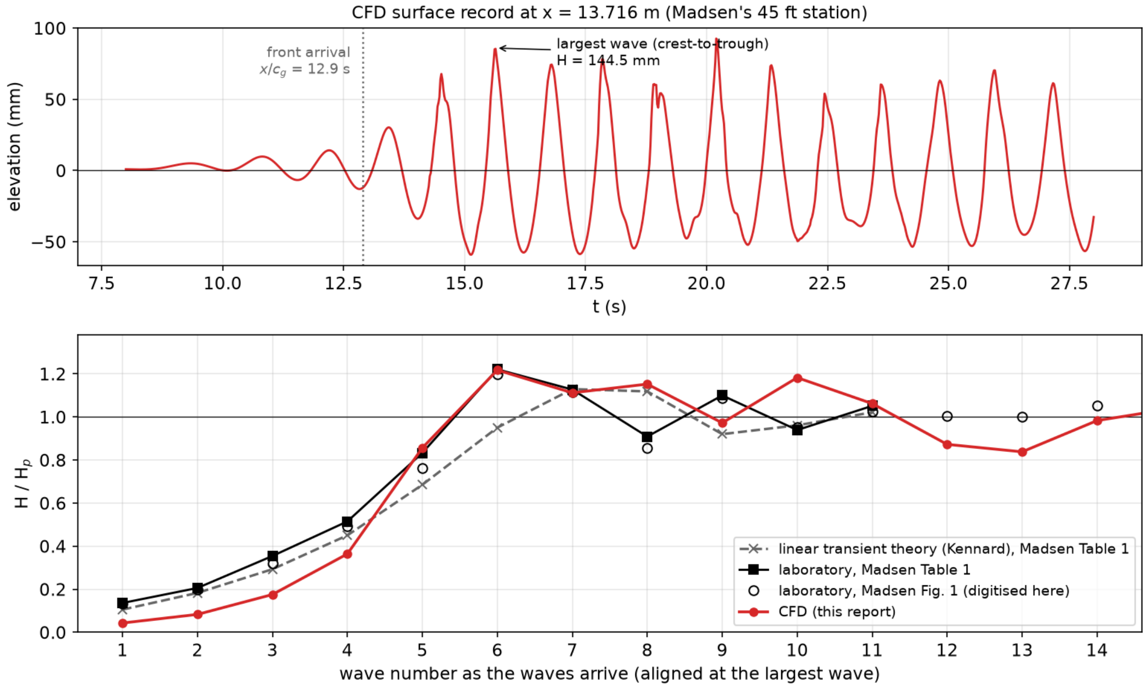

Figure 3. Top: CFD surface record at Madsen's 45 ft station - front arrives at the group-velocity time, the largest wave follows a few periods later, then the burst settles. Bottom: the starting transient wave by wave, H/Hp: laboratory (squares: Table 1; circles: digitised Fig. 1 record), linear transient theory (dashed), CFD (red). Overshoot: laboratory 1.20-1.22, CFD 1.22, linear theory 1.13.

4.4 The amplitude budget along the flume

Figure 4 places the absolute wave heights honestly. The CFD delivers 100.5% of theory at the paddle, 94-95% at 4.2 wavelengths, 80% at Madsen's station (7.2 wavelengths) and 91% at 8.4 wavelengths. Two mechanisms overlay: numerical dissipation of a steep (H/L = 0.074) wave over a long propagation path on this single grid, and a spatial modulation of the height envelope by the free second harmonic - the parasitic wave Madsen's own second-order wavemaker theory predicts for a sinusoidally driven piston. The measured second-harmonic content varies along the flume (a_2/a_1 = 0.15-0.24), which is that beat made visible, and is why the decay is not monotonic. The laboratory bracket puts these numbers in context: Madsen's paddle leaked around its edges, delivering 65% (chart-record run) to 71% (tabulated run) of theory at the same station - his own attribution - while an ideal fluid would carry 100%. A CFD flume with no leakage should sit between the leaky laboratory and the ideal theory, and does. For generation design the operative numbers are the near-field ones; the far-field decay is quantified here so that no one mistakes a 7-wavelength propagation figure for a generation error.

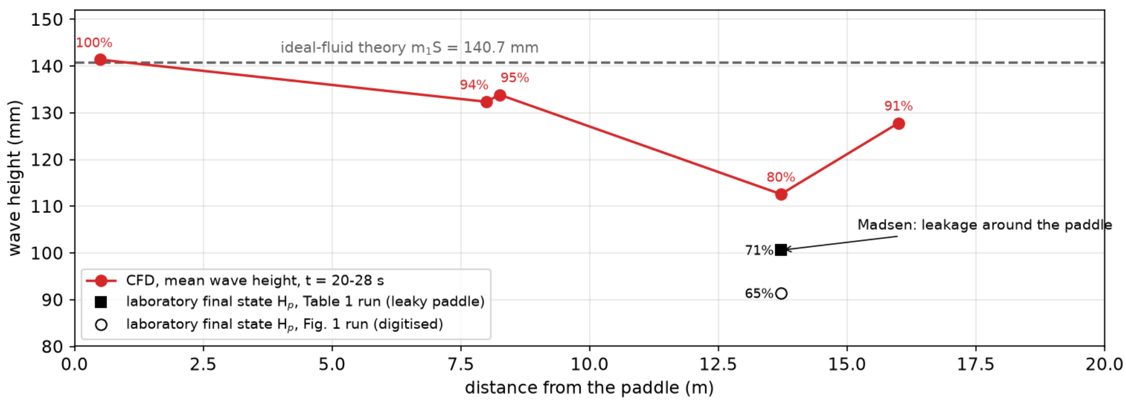

Figure 4. Mean wave height along the flume (t = 20-28 s) against ideal-fluid theory and against the laboratory final state at 45 ft. The CFD (no paddle leakage) sits between the leaky laboratory and the ideal theory; the non-monotonic profile is the free-second-harmonic beat over numerical dissipation.

5. Limits

One kh was tested (kh = 1.508, h/L = 0.240) - the transfer-function curve is validated at a single point, chosen because it carries the best-documented transient benchmark; other regimes inherit the theory, not this measurement. The flume is 2D: transverse modes, three-dimensional paddle effects and physical leakage are absent by construction - which is also what makes the leakage-free comparison against theory meaningful. The amplitude decay of Section 4.4 (to 80% at 7.2 wavelengths at steepness 0.074) is a property of this grid and steepness, quantified so it can be budgeted; a grid-refinement study of the far field was not part of this report. The start protocol (2T ramp) differs from the laboratory's abrupt start, affecting the rise flank and the absolute wave numbering of the burst, as stated in Section 4.3. The Benjamin-Feir instability that Madsen flags for the laboratory tail at this h/L is present in principle in the simulation but has only numerical noise to grow from over 7 wavelengths; the ring-down comparison carries that asymmetry. Sub-millimetre surface detail is below the 5 mm vertical cell size and is not claimed.

6. Reproducibility

The case is built entirely from standard OpenFOAM v2512 components (interFoam, waveMaker piston boundary, displacementLaplacian mesh motion, shallowWaterAbsorption); the full case set-up, gauge records, the digitisation script and the digitised extrema of Madsen's Fig. 1, and the scripts that regenerate every figure and this report are available from the author on request. The laboratory reference is public: Madsen (1970) as CERC Reprint 4-71, approved for public release; Table 1 values were transcribed from p. 596 of that reprint and the chart record digitised from its p. 590 at 300 dpi, with the elevation scale cross-checked against the printed axis ticks (linear to 0.5%) and the time scale against both the printed 1.16 s marker and the steady-train period (consistent within 4%; the wave-height analysis uses only the extrema, not the time scale).

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